Jet and Rocket Propulsion

AE4451

LECTURE 18

Sedina Tsikata – February 20, 2023

Overview

- what we saw last time
  - rockets
    - introduction: types of rocket systems, components, propellants
    - thrust and impulse
    - vehicle acceleration

- today:
  - vehicle acceleration
  - cycle analysis for chemical rockets
  - thrust coefficient and key dependencies

Rocket propulsion systems: vehicle acceleration

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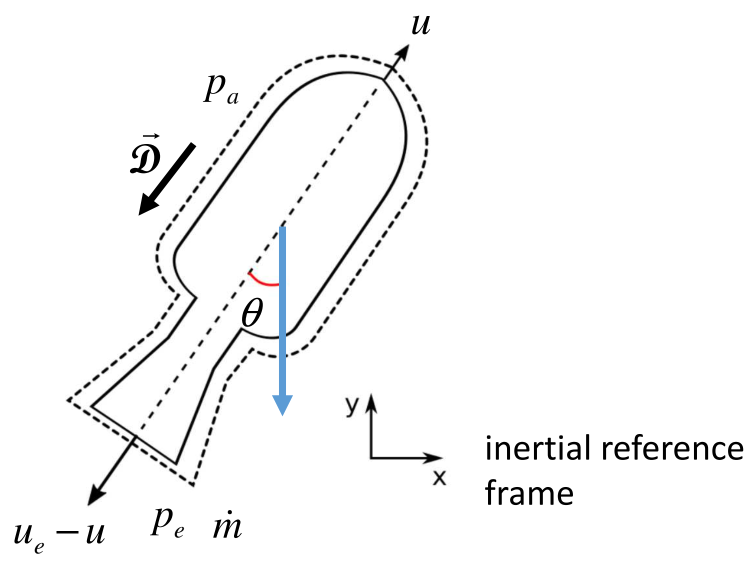
Velocity increment

ideal rocket equation

$$\Delta u = -u_{eq} \ln \frac{m_{final}}{m_{initial}} = u_{eq} \ln \frac{m_{inital}}{m_{final}}$$



mass ratio

$$\mathcal{R} \equiv \frac{m_o}{m_b} \qquad = \frac{\text{initial mass}}{\text{final mass}}$$

$$m_b = m_0 - m_p = \text{burnout / final / dry mass}$$

$$m_p = \text{propellant mass}$$

lower mass ratio for same Δu : higher efficiency

and since $I_{sp} = \dfrac{u_{eq}}{g_e}$

we can also write $\Delta u = I_{sp} g_e \ln \mathcal{R}$

ideal case

- Δu = function of Δm, I_{sp}
- not dependent on burn time

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Rocket equation

more generally, $\int du = \int \frac{\dot{m} u_{eq}}{m} dt - \int g \cos\theta dt - \int \frac{\mathcal{D}}{m} dt - \ldots \text{other terms}$

$$\Delta u = \Delta u_{propulsion} - \Delta u_{\substack{gravity \\ loss}} - \Delta u_{\substack{drag \\ loss}} - \Delta u_{steering}$$

if u_{eq} still constant

$$u_{eq} \ln \mathcal{R}$$

$$\mathcal{R} \equiv \frac{m_o}{m_b}$$

- comes from having to lift mass against a gravity field
- reduce by dropping m early
  - drop propellant fast (**short burn times**)
  - drop "dead" mass (**staging**)

$$\mathcal{D} \sim C_D \frac{1}{2} \rho u^2 A_{ref}$$

C_D = drag coefficient

- want low u & time where density is high (low alt.)
- lower $C_D = C_D(M, \text{shape})$
- also want to minimize vehicle stresses

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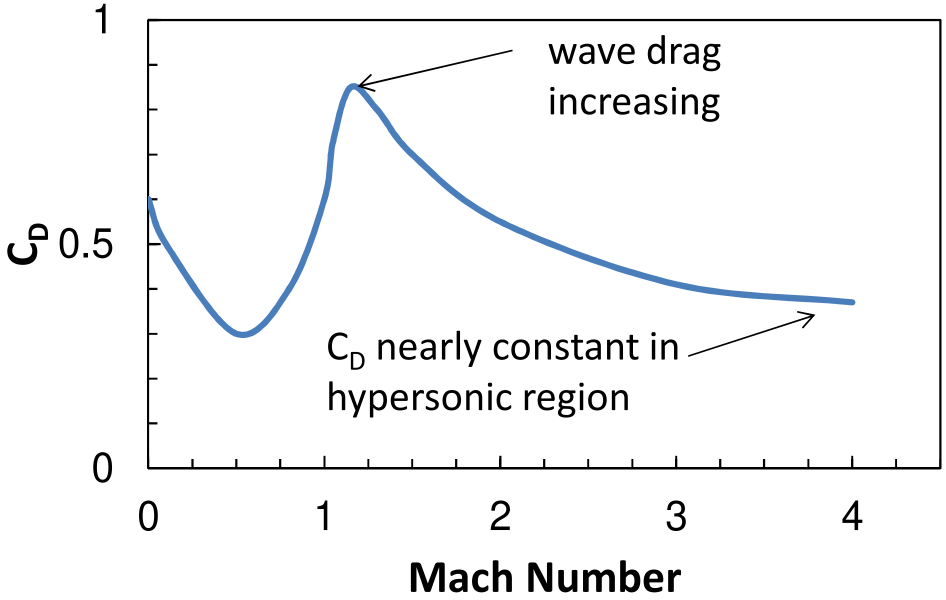
Drag coefficient

example: drag coefficient for a "slender" shaped rocket

(also depends on AOA, angle of attack)



adapted from *Space Propulsion Analysis and Design* (1995), Fig. 2.13

$$c_{Dw} \propto \frac{1}{\sqrt{(M^2-1)}}$$ wave drag coefficient

notion of "sound barrier"

- pressure (sound) waves propagate speed of sound *a*
- at sea level: *a* ~ 1239 km/h
- for *u* > *a*: waves no longer propagate ahead of vehicle, but rather create "wake" that follows vehicle
- sudden increase in aerodynamic drag associated with formation of shock waves

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LEO (low-earth orbit) budgets

for various launch vehicles (in m/s):

velocity at orbital insertion

	u_{LEO}	Δu_{grav}	Δu_{drag}	$\Delta u_{steering}$	Δu_{rot}	$\Delta u_{prop} = \sum \Delta u_i$
Ariane A-44L	7802	1576	38	135	-413	9138
Atlas I	7946	1395	167	110	-375	9243
Delta 7925	7842	1150	33	136	-347	8814
Space Shuttle	7794	1222	358	107	-395	9086
Saturn V	7798	1534	243	40	-348	9267
Titan IV/Centaur	7896	1442	65	156	-352	9207

adapted from Humble, Space Propulsion Analyis and Design

- ascent Δv between 8.8 – 9.3 m/s
- gravity is largest loss term (retards vehicle)
  - shallower flight path angles, shorter burn times (higher acceleration) to reduce effect
- Earth rotation affects velocity budget for orbit insertion: can be taken advantage of (negative values in table)
- steering losses can be minimized by direct flight into circular or elliptical orbit: but gravity loss term more significant

Rocket propulsion systems: vehicle acceleration

LEO (low-earth orbit) budgets

we previously found an expression for the velocity increment

$$\Delta u = u_{eq} \ln \frac{m_{inital}}{m_{final}} \qquad \frac{\Delta u_{prop}}{u_{eq}} = \ln \mathcal{R}$$

given that typically, $u_{LEO} \sim 2.5 - 3.5 \times u_{eq,chem}$

$$\Rightarrow \mathcal{R}_{LEO} \sim e^{2.5-3.5}$$

$\sim 12 - 35$: this translates to a launch mass of which > 90% is propellant

Rocket propulsion systems

Chemical rocket ideal cycle

As with air-breathing engines, we are interested in predicting rocket performance (thrust, impulse,…)

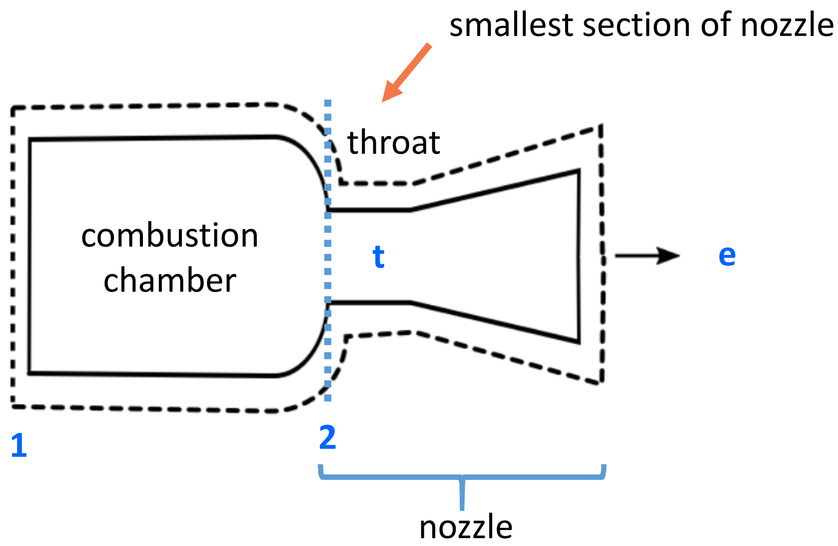
- start by considering simplified, ideal rocket cycle with
  1) combustion chamber
  2) nozzle

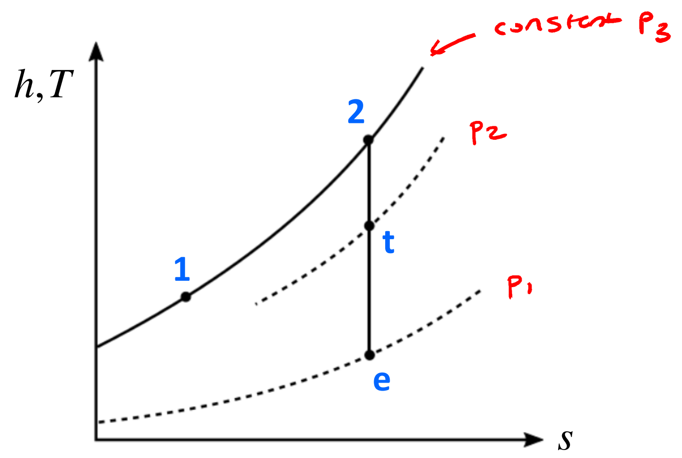
use idealizing assumptions:

- steady flow, thermally and calorically perfect gas, constant properties (MW, γ)
- chemical reaction equivalent to constant pressure heating (reversible)
- nozzle expansion is 1D, reversible and adiabatic (isentropic)

Rocket propulsion systems

Chemical rocket ideal cycle





Rocket propulsion systems

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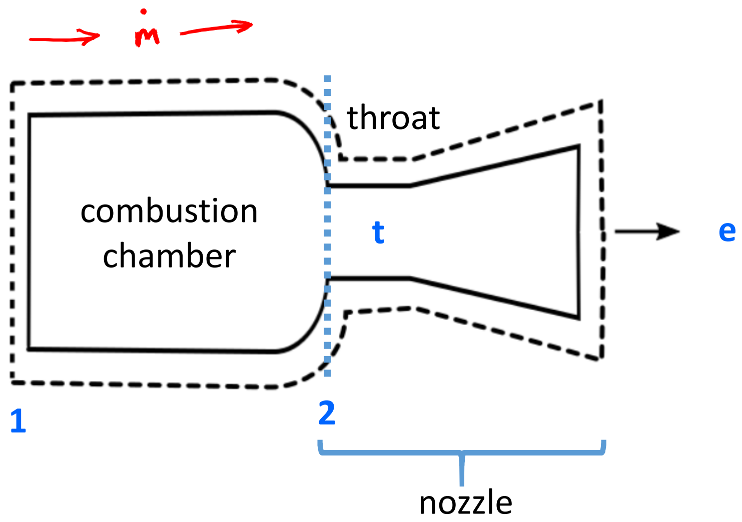
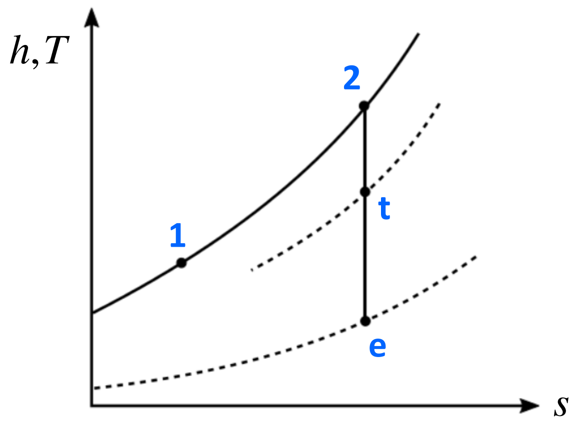
Chemical rocket ideal cycle: combustion chamber





- from energy conservation (adiabatic, no work)

$$\dot{m}\Delta h_R = \dot{m}\left(h_{o2} - h_{o1}\right) = \dot{m}c_p\left(T_{o2} - T_{o1}\right)$$

$$T_{o2} = T_{o1} + \Delta h_R / c_p$$

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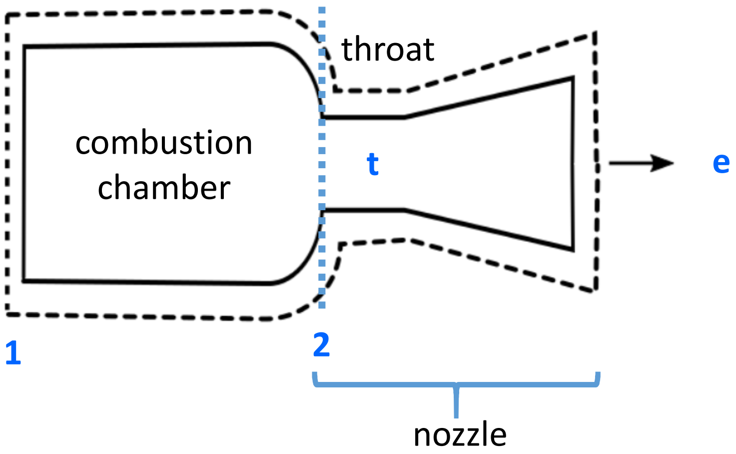
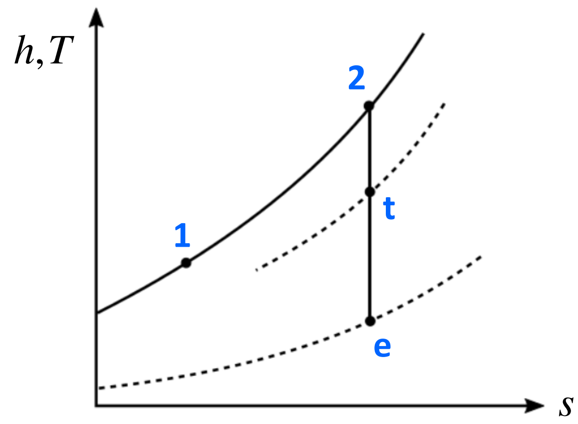
Chemical rocket ideal cycle: nozzle





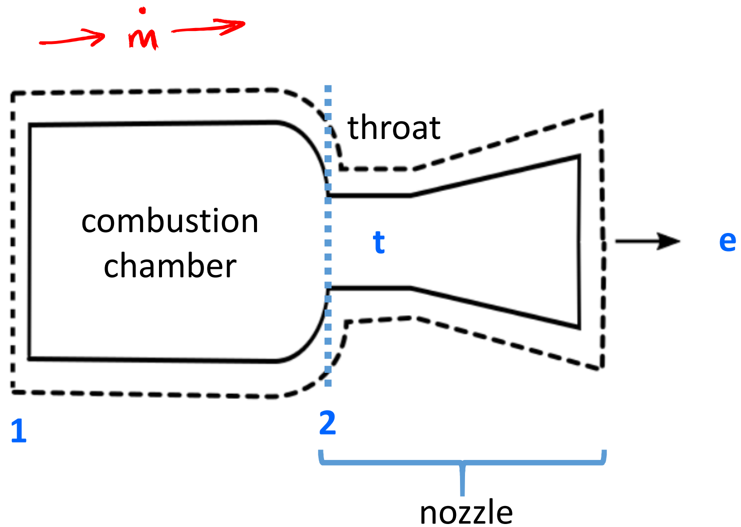
- from energy conservation (adiabatic, no work)

matching stagnation enthalpy at 2 and e $\quad \dot{m}h_{o2} = \dot{m}\left(h_e + u_e^2/2\right) \quad \Rightarrow u_e = \sqrt{2\left(h_{o2} - h_e\right)} = \sqrt{2c_p T_{o2}\left(1 - T_e/T_{o2}\right)}$

reversible $\quad u_e = \sqrt{\dfrac{2\gamma}{\gamma-1} R T_{o2}\left[1 - \left(\dfrac{p_e}{p_{o2}}\right)^{\gamma-1/\gamma}\right]}$

Rocket propulsion systems

Chemical rocket ideal cycle: nozzle



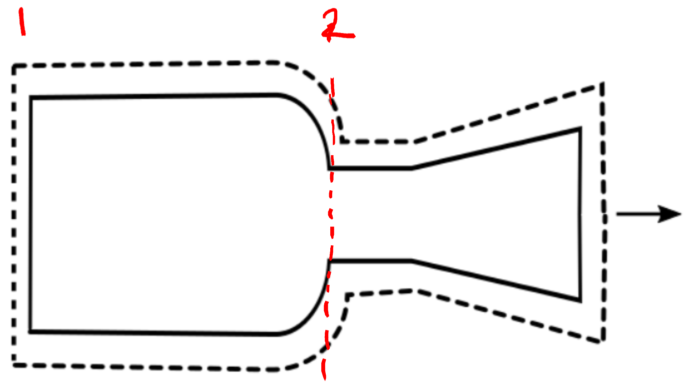
- can we increase the mass flow rate indefinitely across the nozzle?

- mass flow rate $\dot{m} = \rho u A_t$

- density ρ changes as u increases for real compressible fluids: how does this affect the achievable mass flow rate?

Rocket propulsion systems

Chemical rocket ideal cycle: nozzle



$$\dot{m} = \rho u A_t$$

1. rewrite u

$$u = Ma = M\sqrt{\gamma RT}$$ (a = sound speed, M = Mach number)

2. use state equation to express ρ $\rho = \dfrac{P}{RT}$

3. use isentropic flow relations $\dfrac{p_0}{p} = \left(\dfrac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$ $\dfrac{T_0}{T} = 1 + \dfrac{\gamma - 1}{2}M^2$

3. after substitution, regrouping terms in $\dot{m} = \rho u A_t$

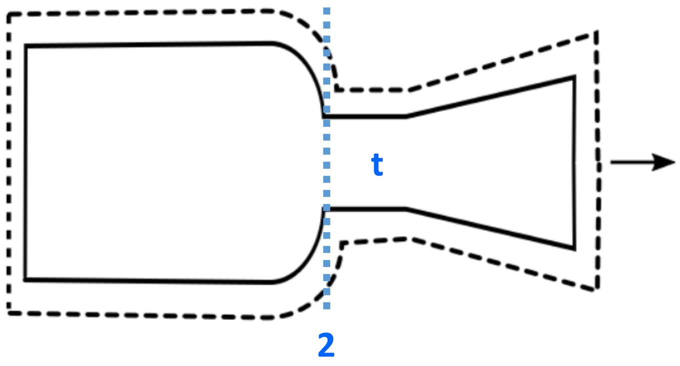
new form for mass flow rate in terms of M and stagnation properties entering throat

$$\dot{m} = \frac{A_t p_o}{\sqrt{T_o}}\sqrt{\frac{\gamma}{R}}M\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

Rocket propulsion systems

Chemical rocket ideal cycle: nozzle

$$\dot{m} = \rho u A_t$$



- maximum flow rate is when M = 1 **"choked flow"**

- the maximum mass flow rate through the system occurs when the flow is choked at the smallest area (throat)

$$\dot{m} = \frac{A_t p_o}{\sqrt{T_o}} \sqrt{\frac{\gamma}{R}} M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

lowering downstream pressure will have no effect on the flow rate

$$\dot{m}_{choked} = A_t \frac{p_o}{\sqrt{RT_o}} \sqrt{\gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

max.

$$= \quad \dot{m}_{choked} = A_t \frac{p_o}{\sqrt{\left(\bar{R}/MW\right) T_o}} \sqrt{\gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

stagnation properties at 2

R = specific gas constant

$\bar{R}$ = molar gas constant

MW = molar mass

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Maximizing specific impulse and thrust

$\gamma = 1.2$ $\quad c_p/c_v = \gamma$

- already saw $I_{sp} \sim u_{eq} \sim u_e$

  how to get high u_e?

  - low MW propellant
  - high p_o/p_e (high combustion chamber pressure and large nozzle area ratio)
  - high T_o (high $\Delta h_R/c_p$, chemical energy)

$$u_e = \sqrt{\frac{2\gamma}{\gamma-1}\frac{\bar{R}}{MW}T_{o2}\left[1-\left(\frac{p_e}{p_{o2}}\right)^{\gamma-1/\gamma}\right]}$$

$$T_{o2} = T_{o1} + \Delta h_R / c_p$$

- thrust ~ $\dot{m}u_e$

  how to get large thrust?

  - large throat area, A_t
  - high chamber p_o
  - low combustion T_o
  - high propellant MW

$$\dot{m} = \frac{p_{o2}}{\sqrt{\bar{R}T_{o2}/MW}} A_t \sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

notice how some of these objectives are in opposition

Rocket propulsion systems: more parameters

Thrust coefficient c_τ

definition $$c_\tau \equiv \frac{\tau}{A_t p_o}$$

exhaust area

* $$\tau = \dot{m}u_e + (p_e - p_a)A_e$$ also steady thrust

can combine with ideal nozzle findings $$u_e = \sqrt{\frac{2\gamma}{\gamma-1}RT_o\left[1-\left(\frac{p_e}{p_o}\right)^{\gamma-1/\gamma}\right]}$$ $$\dot{m}_{choked} = A_t\frac{p_o}{\sqrt{RT_o}}\sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

to define ideal thrust coefficient

$$c_{\tau,ideal} = \gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}\left[1-\left(\frac{p_e}{p_o}\right)^{\gamma-1/\gamma}\right]} + \left(\frac{p_e}{p_o}-\frac{p_a}{p_o}\right)\frac{A_e}{A_t}$$

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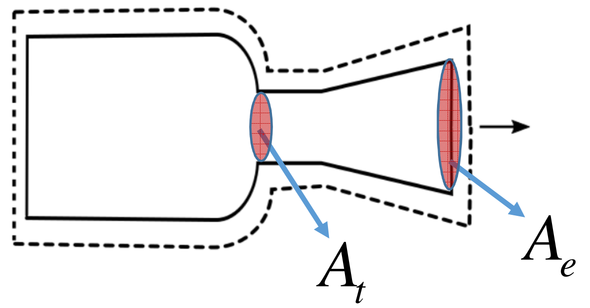
Ideal thrust coefficient $c_{\tau,ideal}$

$$c_{\tau,ideal} = \gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}\left[1-\left(\frac{p_e}{p_o}\right)^{\gamma-1/\gamma}\right]} + \left(\frac{p_e}{p_o}-\frac{p_a}{p_o}\right)\frac{A_e}{A_t}$$

$$\varepsilon = \frac{A_e}{A_t}$$



- ideal thrust coefficient only a function of $\gamma,\ \varepsilon,\ p_a/p_o$

  A_e/A_t

$$p_e/p_o = f(\varepsilon)$$

- note that

$$c_\tau \neq f(T_o, MW)$$ a nozzle characteristic

- the thrust coefficient depends mainly on the pressure distribution in the chamber $c_\tau \equiv \dfrac{\tau}{A_t p_o}$

Rocket propulsion systems: more parameters

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Characteristic velocity c*

- we can define a similar parameter for the combustor

definition $$c^* \equiv \frac{p_o A_t}{\dot{m}}$$

- we can define an ideal characteristic velocity

using $$\dot{m}_{choked} = A_t \frac{p_o}{\sqrt{RT_o}} \sqrt{\gamma \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}} \Rightarrow c^*_{ideal} = \sqrt{\frac{1}{\gamma} \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{\gamma-1}} \frac{\bar{R}}{MW} T_o}$$

combustion process characteristic

- note that

$$c^*_{ideal} = f(T_o, MW, \gamma)$$ a propellant combustion property

- also we can now write

$$c_\tau c^* = \frac{\tau}{p_o A_t} \frac{p_o A_t}{\dot{m}} = \frac{\tau}{\dot{m}} \Rightarrow c_\tau c^* = u_{eq}$$

Rocket propulsion systems: more parameters

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Characteristic velocity c*

c_τ = thrust coefficient

- revisiting our table of liquid bipropellants

Oxidizer	BP/FP (°C)	Fuel	BP/FP (°C)	Combustor Temperature (K)	Bulk Avg. Density (g/cm³)	C* (m/s)	Isp (s)	$u_{eq} = ?$ (m/s)
O_2	-183/-218	H_2	-253/-259	3010	0.3	2420	390	3830
O_2		RP-1	~210/-50	3680	1.0	1810	300	2940
O_2		UDMH	63/-58	3600	1.0	1860	310	
O_2		NH_3	-33/-78	3080	0.9	1800	295	
F_2	-188/-220	H_2		3960	0.5	2560	410*	4020
F_2		Hydrazine	113/1.4	4680	1.3	2210	363*	
N_2O_4	21/-12	MMH	86/-53	3390	1.2	1750	288*	
N_2O_4		RP-1		3450	1.3	1650	275	2700

Optimum performance; 1000psia (6.94MPa) combustor; $p_e=p_a$=14.7 psia (1 atm)

UDMH=Unsymmetrical dimethyl hydrazine $(CH_3)_2NNH_2$ **Hydrazine**=N_2H_4

MMH=Monomethyl hydrazine $CH_3NH\text{-}NH_2$ **NH_3**=Ammonia

*__Hypergolic Mixture__ (ignites on contact)

$$c_\tau c^* = u_{eq}$$

- generally $c^* < u_{eq}$
  - because with well designed system $c_\tau > 1$

Rocket propulsion systems: more parameters

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Thrust coefficient: momentum vs. pressure

- let's re-examine our expression for the thrust coefficient

$$c_{\tau,ideal} = \gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}\left[1-\left(\frac{p_e}{p_o}\right)^{\gamma-1/\gamma}\right]} + \left(\frac{p_e}{p_o}-\frac{p_a}{p_o}\right)\frac{A_e}{A_t}$$

$\dot{m}u_e / p_o A_t$ — contribution to thrust by **exit velocity**/momentum

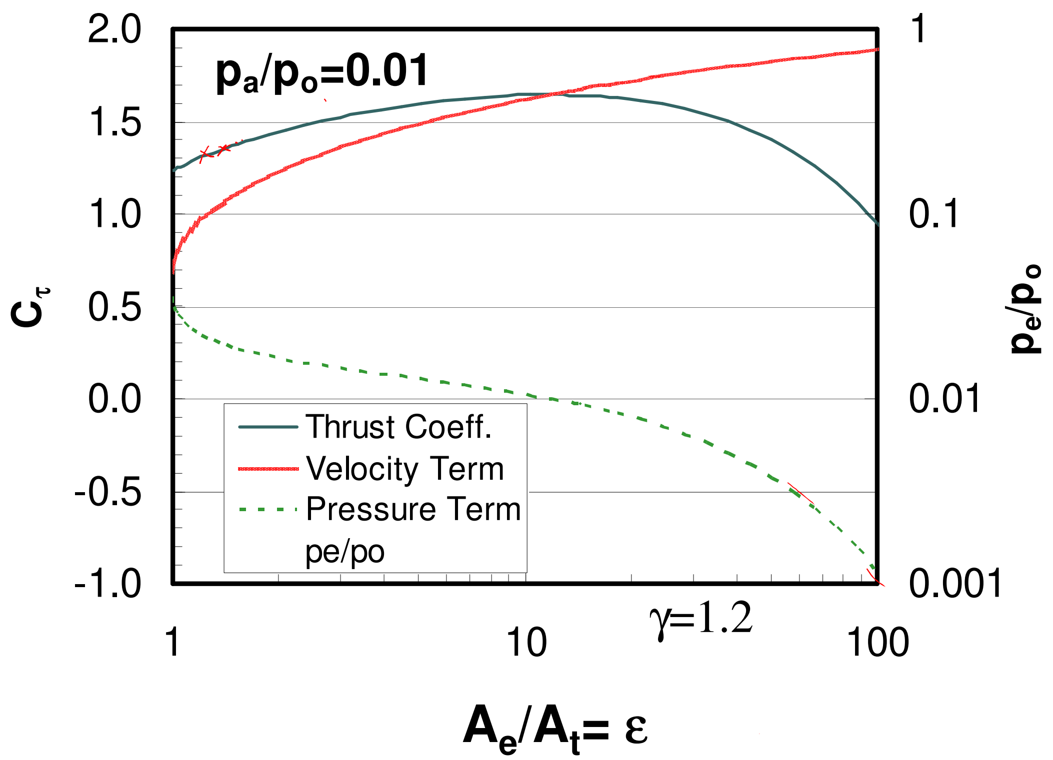
$(p_e - p_a)A_e / p_o A_t$ — contribution to thrust by **exit pressure**

Rocket propulsion systems: more parameters

Comparison of terms

- different nozzle designs

$$c_{\tau,ideal} = \gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}\left[1-\left(\frac{p_e}{p_o}\right)^{\gamma-1/\gamma}\right]} + \left(\frac{p_e}{p_o}-\frac{p_a}{p_o}\right)\frac{A_e}{A_t}$$



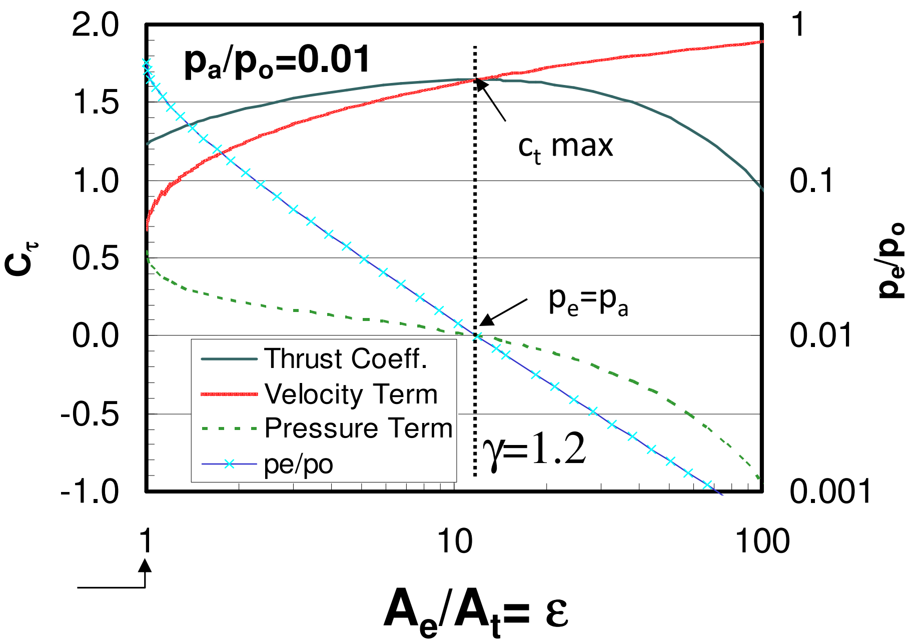
- velocity term always provides thrust (+)
- pressure term can increase or decrease thrust

Rocket propulsion systems: more parameters

Comparison of terms

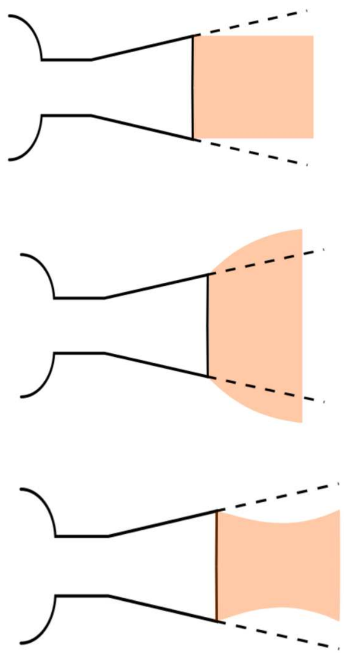
- exit vs ambient pressure

$$c_{\tau,ideal} = \gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}\left[1-\left(\frac{p_e}{p_o}\right)^{\gamma-1/\gamma}\right]} + \left(\frac{p_e}{p_o}-\frac{p_a}{p_o}\right)\frac{A_e}{A_t}$$



maximum thrust is achieved (for fixed p_o, p_a) if perfectly expanded

i.e. exhaust pressure matches ambient pressure



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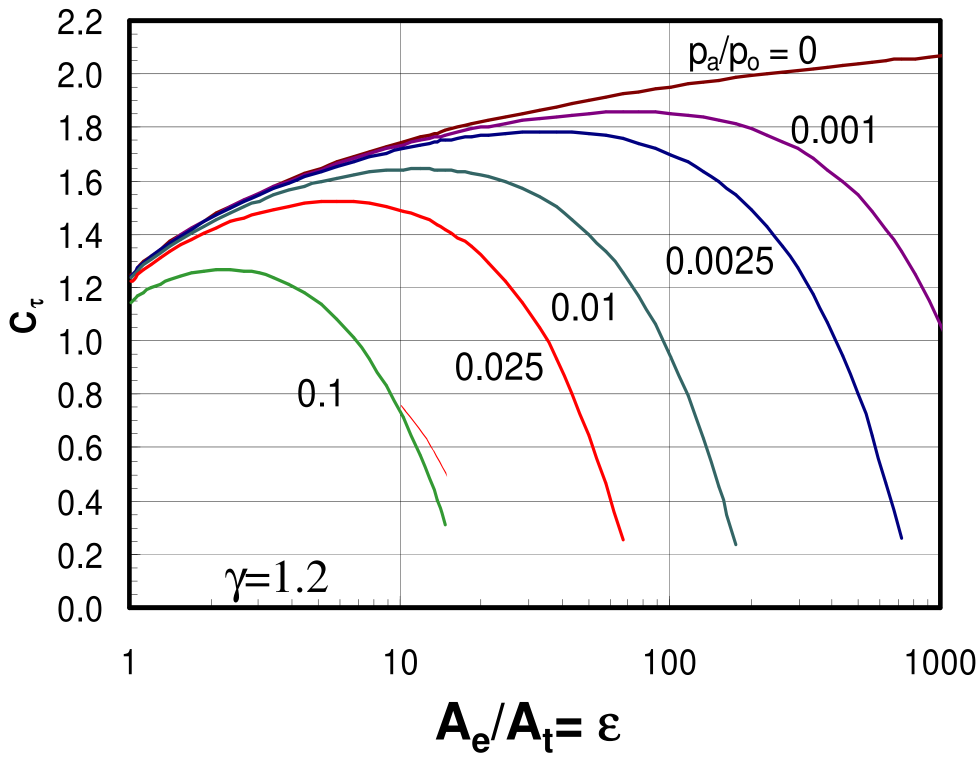
Effect of ambient pressure on c_τ



$$c_{\tau,ideal} = \gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}\left[1-\left(\frac{p_e}{p_o}\right)^{\gamma-1/\gamma}\right]} + \left(\frac{p_e}{p_o}-\frac{p_a}{p_o}\right)\frac{A_e}{A_t}$$

can get higher thrust coefficient by:

- reducing ambient pressure
- increasing rocket pressure

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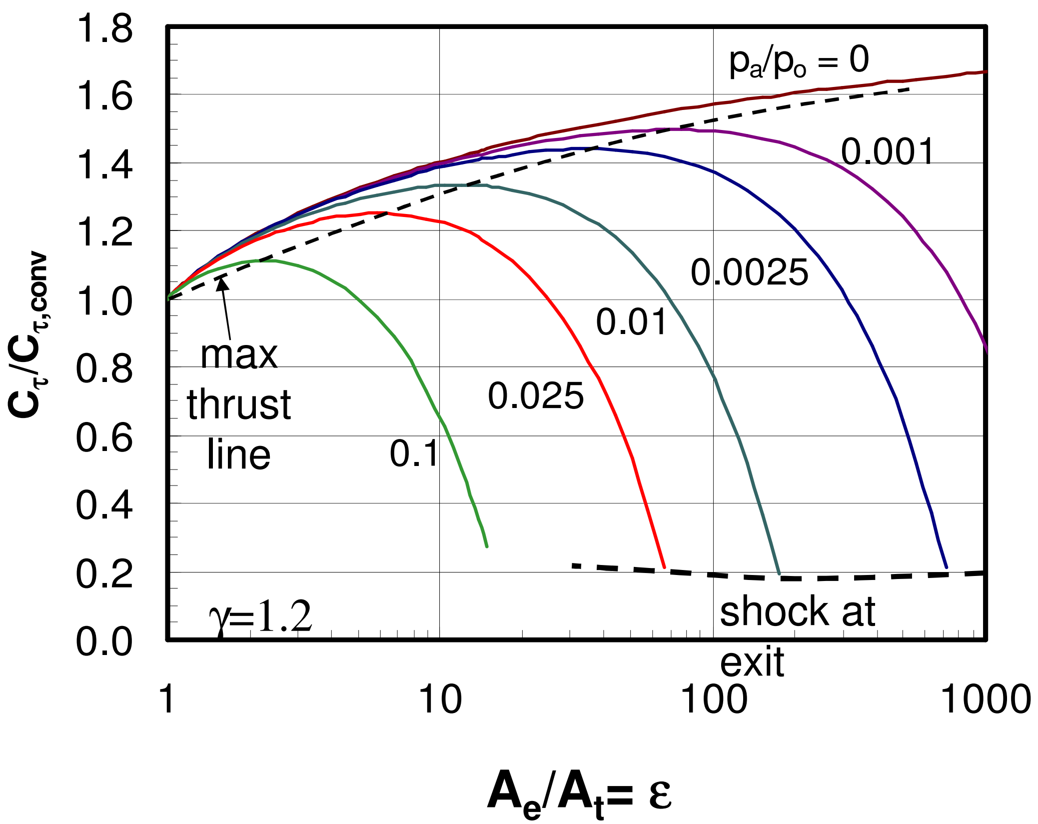
Effect of ambient pressure on c_τ

normalize by converging nozzle



- with small p_a/p_o

  need large ε for optimum c_τ

- ε for optimum c_τ (or Isp) varies with altitude (p_a)

for single nozzle, best ε is closer to low altitude optimum value